Light at the metal border

Specifics of light reflection from metal surface is driven by the presence of free electrons. Induced vibrations of free electrons under external incident e/m wave can occur even in thin border layer as free $\bar{e}$ density is $10^{22}\ \text{cm}^{-3}$.

How we use this property?

We are making mirrors.

Partial penetration of light into the metal creates conduction current, thus release of heat - light absorption.

The larger is conductivity, the smaller is portion of light that penetrates into the metal, hence, smaller the absorption.

We can employ Maxwell's equations:

$$c^2\left[\bar{\nabla}\bar{B}\right] - \frac{\partial \bar{E}}{\partial t} = \frac{\bar{j}}{\varepsilon_0}$$

Here j is the density of conductive currents.

From Ohm's law in differential form:

$$\bar{j} = \sigma\bar{E}$$

$$\varepsilon_0 c^2\left[\bar{\bar{\nabla}}\bar{B}\right] = \sigma\bar{E} + \varepsilon\varepsilon_0\dot{\bar{E}} = \sigma\bar{E} + \varepsilon\varepsilon_0 i\omega\bar{E} = \qquad \begin{cases} \bar{E} = \bar{E}_0 e^{i\omega t} \\ \dot{\bar{E}} = \bar{E}_0 i\omega e^{i\omega t} \end{cases}$$

$$= \tilde{\varepsilon}\varepsilon_0 i\omega \bar{E}$$

$$\tilde{\varepsilon}\varepsilon_0 i\omega = \sigma + \varepsilon\varepsilon_0 i\omega$$

$$\tilde{\varepsilon} = \varepsilon - \frac{i\sigma}{\omega\varepsilon_0} =$$ $\tilde{\varepsilon}$ - complex dielectric constant

$$= \varepsilon - i\varepsilon''$$

Thus,

$$c^2\left[\bar{\nabla}\bar{B}\right] = \tilde{\varepsilon}\varepsilon_0 i\omega \bar{E}$$

This equation formally is similar to one used for dielectrics. Thus one can use similar derivations and conclusions, keeping in mind multiplication on i.

One can use complex refractive index:

$$\tilde{n}^2 = \varepsilon - i\varepsilon''$$

assuming

$$\tilde{n} = n - i\varkappa$$

$$n^2 + \varkappa^2 = \varepsilon$$

$$2n\varkappa = \varepsilon''$$

For metals n is called main refractive index and $\varkappa$ is called main damping constant.

Since for dielectrics $k = \frac{\omega}{c} n$

For metals $k^2 = \frac{\omega^2}{c^2}\tilde{\varepsilon}^2 = k\frac{\omega^2}{c^2}(\varepsilon - i\varepsilon'')$

In absorbing media the wave vector is a complex function, hence:

$$\bar{k} = \bar{k}' - i\bar{k}''$$

$$\Downarrow$$

$$\begin{cases} \bar{k}'^2 - \bar{k}''^2 = \frac{\omega^2}{c^2}\varepsilon \\ 2(\bar{k}'\bar{k}'') = \frac{\omega^2}{c^2}\varepsilon'' \end{cases} \qquad (1)$$

What are the directions of $\bar{k}'$ and $\bar{k}''$?

If we write the wave:

$$E_0 e^{i(\omega t - \bar{k}\bar{r})} = E_0 e^{i(\omega t - \bar{k}'\bar{r} + i\bar{k}''\bar{r})} =$$

$$= E_0 e^{-\bar{k}''\bar{r}} e^{i(\omega t - \bar{k}'\bar{r})}$$

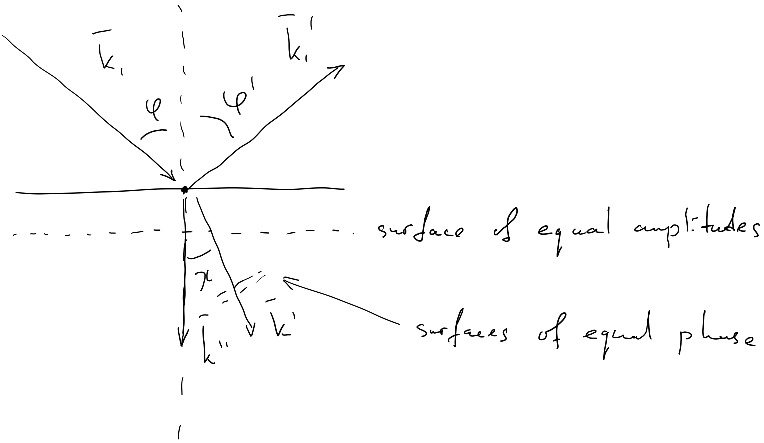
1) k'' indicates the direction of amplitude decrease

2) k' indicates the direction of propagation of surfaces of equal phase.

It is important to note, $\bar{k}'$ and $\bar{k}''$ coincide in propagation. The only case it will be $\frac{\pi}{2}$ phase difference, when $\varepsilon''=0$.

That means there is no damping. In practice, such picture is impossible for infinite medium, but can be realized on the border.

Reflection and refraction of light on metal



① Assuming similar boundary conditions

$$k_{1x} = k'_{1x} = k_x$$

$$\begin{cases} k_{1x} = \frac{\omega}{c} n_1 \sin\varphi \\ k'_{1x} = \frac{\omega}{c} n_1 \sin\varphi' \end{cases} \Rightarrow \varphi = \varphi'$$

② Since $k_{1x} = k'_{1x} = k_x$, then tangential component should be real number. Thus k'' has to be

$\perp$ to metal surface.

③ Since $2(\bar{k}'\bar{k}'') = \frac{\omega^2}{c^2}\varepsilon'' > 0$, then $\angle$ k' and k'' $< \frac{\pi}{2}$.

k' indicates the direction of wave propagation and is perpendicular to the surfaces of equal phases.

χ is called real angle of refractive index